

PEMBERTON HEATH

(615) 461-5888

[pheath@kippnashville.org](mailto:pheath@kippnashville.org)

Office Hours: 10:00 a.m. – 2:00 p.m via text or google hangouts



## Pre-Calculus

### Learning Packet Overview

In this two week review packet, students will review the most high leverage concepts and skills from our study of trigonometry. Students will focus on aspects of trigonometry most essential for success on the ACT exam and in AP Calculus. The learning packet will offer a “digital track” where students will complete review via daily Khan Academy videos followed by daily practice. For students who prefer to work offline, there will be a “paper and pencil” track where students will read textbook excerpts and complete daily review problems in this packet.

### Necessary Materials

- Calculator (email Ms. Heath if you do not have your calculator at home for options on downloading simulators online for free)
- Pencil and paper (even if performing digital track, for scratch work!)

### How students will be successful in Pre-Calculus

Students will be successful if they:

- Watch the review video or read the review reading prior to completing problems
- If reading the textbook excerpt, complete any worked examples alongside of the textbook
- Attempt the review problems, and note which problems are difficult/what questions arise
- Utilize office hours to ask questions. If unable to ask them live during office hours, email Ms. Heath your questions.

### How caregivers can help students be successful

Caregivers can help students be successful by:

- Encourage students to commit 30 minutes M-F to reviewing pre-calculus skills
- Cheer on students who are working on difficult math!
- Encourage students to reach out for help early and often

<b>Digital Track Overview</b>	<b>Paper and Pencil Overview</b>
<ul style="list-style-type: none"> <li>• Assignments will be posted regularly on google classroom.</li> <li>• Google classroom posts will direct students to Khan Academy for videos/review problems</li> </ul>	<ul style="list-style-type: none"> <li>• Each topic will consist of a short review reading/worked example followed by practice problems</li> <li>• Work should be completed on a separate sheet of paper. Clearly indicate which problems you are working on by noting the page and topic.</li> </ul>
<b>Topics Covered</b>	
<ul style="list-style-type: none"> <li>• Right triangle trigonometry</li> <li>• Radian and degree measure</li> <li>• Evaluating trigonometric functions at special angles</li> <li>• Evaluating inverse trigonometric functions at special angles</li> <li>• Recognizing graphs of sine and cosine</li> <li>• Sketching graphs of sine and cosine by identifying transformations from the parent function</li> </ul>	

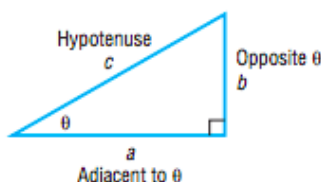
### Right triangle trigonometry

#### DEFINITION

The six ratios of the lengths of the sides of a right triangle are called **trigonometric functions of acute angles** and are defined as follows:

Function Name	Abbreviation	Value	Function Name	Abbreviation	Value
sine of $\theta$	$\sin \theta$	$\frac{b}{c}$	cosecant of $\theta$	$\csc \theta$	$\frac{c}{b}$
cosine of $\theta$	$\cos \theta$	$\frac{a}{c}$	secant of $\theta$	$\sec \theta$	$\frac{c}{a}$
tangent of $\theta$	$\tan \theta$	$\frac{b}{a}$	cotangent of $\theta$	$\cot \theta$	$\frac{a}{b}$

Figure 20



As an aid to remembering these definitions, it may be helpful to refer to the lengths of the sides of the triangle by the names *hypotenuse* ( $c$ ), *opposite* ( $b$ ), and *adjacent* ( $a$ ). See Figure 20. In terms of these names, we have the following ratios:

$$\begin{aligned}
 \sin \theta &= \frac{\text{opposite}}{\text{hypotenuse}} = \frac{b}{c} & \cos \theta &= \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{a}{c} & \tan \theta &= \frac{\text{opposite}}{\text{adjacent}} = \frac{b}{a} \\
 \csc \theta &= \frac{\text{hypotenuse}}{\text{opposite}} = \frac{c}{b} & \sec \theta &= \frac{\text{hypotenuse}}{\text{adjacent}} = \frac{c}{a} & \cot \theta &= \frac{\text{adjacent}}{\text{opposite}} = \frac{a}{b}
 \end{aligned}
 \tag{1}$$

### Practice

#### Skill Building

In Problems 11–20, find the value of the six trigonometric functions of the angle  $\theta$  in each figure.

11. 12. 13. 14. 15. 16. 17. 18. 19. 20.

In Problems 21–24, use identities to find the exact value of each of the four remaining trigonometric functions of the acute angle  $\theta$ .

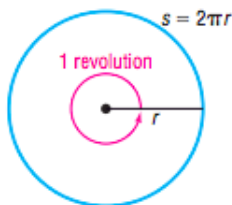
21.  $\sin \theta = \frac{1}{2}$ ,  $\cos \theta = \frac{\sqrt{3}}{2}$     22.  $\sin \theta = \frac{\sqrt{3}}{2}$ ,  $\cos \theta = \frac{1}{2}$     23.  $\sin \theta = \frac{2}{3}$ ,  $\cos \theta = \frac{\sqrt{5}}{3}$     24.  $\sin \theta = \frac{1}{3}$ ,  $\cos \theta = \frac{2\sqrt{2}}{3}$

## Radian and degree measure

### 3 Convert from Degrees to Radians and from Radians to Degrees

Figure 12

1 revolution =  $2\pi$  radians



With two ways to measure angles, it is important to be able to convert from one to the other. Consider a circle of radius  $r$ . A central angle of 1 revolution will subtend an arc equal to the circumference of the circle (Figure 12). Because the circumference of a circle of radius  $r$  equals  $2\pi r$ , we substitute  $2\pi r$  for  $s$  in equation (4) to find that, for an angle  $\theta$  of 1 revolution,

$$\begin{aligned}
 s &= r\theta \\
 2\pi r &= r\theta & \theta &= 1 \text{ revolution}; s = 2\pi r \\
 \theta &= 2\pi \text{ radians} & \text{Solve for } \theta.
 \end{aligned}$$

From this, we have

$$1 \text{ revolution} = 2\pi \text{ radians} \quad (5)$$

Since 1 revolution =  $360^\circ$ , we have

$$360^\circ = 2\pi \text{ radians}$$

Dividing both sides by 2 yields

$$180^\circ = \pi \text{ radians} \quad (6)$$

Divide both sides of equation (6) by 180. Then

$$1 \text{ degree} = \frac{\pi}{180} \text{ radian}$$

Divide both sides of (6) by  $\pi$ . Then

$$\frac{180}{\pi} \text{ degrees} = 1 \text{ radian}$$

We have the following two conversion formulas:\*

$$1 \text{ degree} = \frac{\pi}{180} \text{ radian} \quad 1 \text{ radian} = \frac{180}{\pi} \text{ degrees} \quad (7)$$

**EXAMPLE 4****Converting from Degrees to Radians**

Convert each angle in degrees to radians.

- (a)
- $60^\circ$
- (b)
- $150^\circ$
- (c)
- $-45^\circ$
- (d)
- $90^\circ$
- (e)
- $107^\circ$

- Solution**
- (a)  $60^\circ = 60 \cdot 1 \text{ degree} = 60 \cdot \frac{\pi}{180} \text{ radian} = \frac{\pi}{3} \text{ radians}$
- (b)  $150^\circ = 150 \cdot 1^\circ = 150 \cdot \frac{\pi}{180} \text{ radian} = \frac{5\pi}{6} \text{ radians}$
- (c)  $-45^\circ = -45 \cdot \frac{\pi}{180} \text{ radian} = -\frac{\pi}{4} \text{ radian}$
- (d)  $90^\circ = 90 \cdot \frac{\pi}{180} \text{ radian} = \frac{\pi}{2} \text{ radians}$
- (e)  $107^\circ = 107 \cdot \frac{\pi}{180} \text{ radian} \approx 1.868 \text{ radians}$

Example 4, parts (a)–(d), illustrates that angles that are “nice” fractions of a revolution are expressed in radian measure as fractional multiples of  $\pi$ , rather than as decimals. For example, a right angle, as in Example 4(d), is left in the form  $\frac{\pi}{2}$  radians, which is exact, rather than using the approximation  $\frac{\pi}{2} \approx \frac{3.1416}{2} = 1.5708$  radians. When the fractions are not “nice,” we use the decimal approximation of the angle, as in Example 4(e).

**Practice**

*In Problems 35–46, convert each angle in degrees to radians. Express your answer as a multiple of  $\pi$ .*

35.  $30^\circ$       36.  $120^\circ$       37.  $240^\circ$       38.  $330^\circ$       39.  $-60^\circ$       40.  $-30^\circ$
41.  $180^\circ$       42.  $270^\circ$       43.  $-135^\circ$       44.  $-225^\circ$       45.  $-90^\circ$       46.  $-180^\circ$

*In Problems 47–58, convert each angle in radians to degrees.*

47.  $\frac{\pi}{3}$       48.  $\frac{5\pi}{6}$       49.  $-\frac{5\pi}{4}$       50.  $-\frac{2\pi}{3}$       51.  $\frac{\pi}{2}$       52.  $4\pi$
53.  $\frac{\pi}{12}$       54.  $\frac{5\pi}{12}$       55.  $-\frac{\pi}{2}$       56.  $-\pi$       57.  $-\frac{\pi}{6}$       58.  $-\frac{3\pi}{4}$

### Evaluating trigonometric functions at special angles

$\theta$ (Radians)	$\theta$ (Degrees)	$\sin \theta$	$\cos \theta$	$\tan \theta$	$\csc \theta$	$\sec \theta$	$\cot \theta$
$\frac{\pi}{6}$	$30^\circ$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	2	$\frac{2\sqrt{3}}{3}$	$\sqrt{3}$
$\frac{\pi}{4}$	$45^\circ$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	$\sqrt{2}$	$\sqrt{2}$	1
$\frac{\pi}{3}$	$60^\circ$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{2\sqrt{3}}{3}$	2	$\frac{\sqrt{3}}{3}$

$\theta$ (Radians)	$\theta$ (Degrees)	$\sin \theta$	$\cos \theta$	$\tan \theta$	$\csc \theta$	$\sec \theta$	$\cot \theta$
0	$0^\circ$	0	1	0	Not defined	1	Not defined
$\frac{\pi}{2}$	$90^\circ$	1	0	Not defined	1	Not defined	0
$\pi$	$180^\circ$	0	-1	0	Not defined	-1	Not defined
$\frac{3\pi}{2}$	$270^\circ$	-1	0	Not defined	-1	Not defined	0

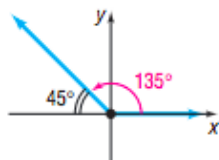
#### EXAMPLE 6

#### Using the Reference Angle to Find the Exact Value of a Trigonometric Function

Find the exact value of each of the following trigonometric functions using reference angles.

(a)  $\sin 135^\circ$       (b)  $\cos 600^\circ$       (c)  $\cos \frac{17\pi}{6}$       (d)  $\tan\left(-\frac{\pi}{3}\right)$

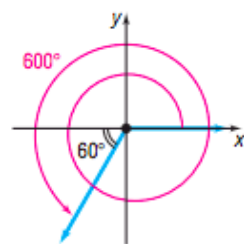
Figure 59

**Solution**

- (a) Refer to Figure 59. The reference angle for  $135^\circ$  is  $45^\circ$  and  $\sin 45^\circ = \frac{\sqrt{2}}{2}$ . The angle  $135^\circ$  is in quadrant II, where the sine function is positive, so

$$\sin 135^\circ = \sin 45^\circ = \frac{\sqrt{2}}{2}$$

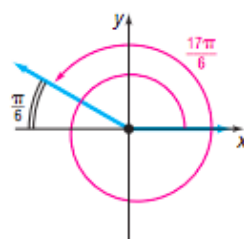
Figure 60



- (b) Refer to Figure 60. The reference angle for  $600^\circ$  is  $60^\circ$  and  $\cos 60^\circ = \frac{1}{2}$ . The angle  $600^\circ$  is in quadrant III, where the cosine function is negative, so

$$\cos 600^\circ = -\cos 60^\circ = -\frac{1}{2}$$

Figure 61



- (c) Refer to Figure 61. The reference angle for  $\frac{17\pi}{6}$  is  $\frac{\pi}{6}$  and  $\cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}$ . The angle  $\frac{17\pi}{6}$  is in quadrant II, where the cosine function is negative, so

$$\cos \frac{17\pi}{6} = -\cos \frac{\pi}{6} = -\frac{\sqrt{3}}{2}$$

- (d) Refer to Figure 62. The reference angle for  $-\frac{\pi}{3}$  is  $\frac{\pi}{3}$  and  $\tan \frac{\pi}{3} = \sqrt{3}$ . The angle  $-\frac{\pi}{3}$  is in quadrant IV, where the tangent function is negative, so

$$\tan\left(-\frac{\pi}{3}\right) = -\tan \frac{\pi}{3} = -\sqrt{3}$$

**Practice**



In Problems 33–40, name the quadrant in which the angle  $\theta$  lies.

33.  $\sin \theta > 0$ ,  $\cos \theta < 0$       34.  $\sin \theta < 0$ ,  $\cos \theta > 0$       35.  $\sin \theta < 0$ ,  $\tan \theta < 0$   
 36.  $\cos \theta > 0$ ,  $\tan \theta > 0$       37.  $\cos \theta > 0$ ,  $\cot \theta < 0$       38.  $\sin \theta < 0$ ,  $\cot \theta > 0$   
 39.  $\sec \theta < 0$ ,  $\tan \theta > 0$       40.  $\csc \theta > 0$ ,  $\cot \theta < 0$

In Problems 41–58, find the reference angle of each angle.

41.  $-30^\circ$       42.  $-60^\circ$       43.  $120^\circ$       44.  $210^\circ$       45.  $300^\circ$       46.  $330^\circ$   
 47.  $\frac{5\pi}{4}$       48.  $\frac{5\pi}{6}$       49.  $\frac{8\pi}{3}$       50.  $\frac{7\pi}{4}$       51.  $-135^\circ$       52.  $-240^\circ$   
 53.  $-\frac{2\pi}{3}$       54.  $-\frac{7\pi}{6}$       55.  $440^\circ$       56.  $490^\circ$       57.  $\frac{15\pi}{4}$       58.  $\frac{19\pi}{6}$

In Problems 59–82, use the reference angle to find the exact value of each expression. Do not use a calculator.

- |  |                          |  |                                       |                           |  |
|--|--------------------------|--|---------------------------------------|---------------------------|--|
|  59. $\sin 150^\circ$ | 60. $\cos 210^\circ$     |  61. $\sin 510^\circ$ | 62. $\cos 600^\circ$ $-\frac{1}{2}$   | 63. $\cos(-45^\circ)$     | 64. $\sin(-240^\circ)$                 |
| 65. $\sec 240^\circ$   | 66. $\csc 300^\circ$     | 67. $\cot 330^\circ$   | 68. $\tan 225^\circ$                  | 69. $\sin\frac{3\pi}{4}$  | 70. $\cos\frac{2\pi}{3}$               |
| 71. $\cos\frac{13\pi}{4}$  | 72. $\tan\frac{8\pi}{3}$ | 73. $\sin\left(-\frac{2\pi}{3}\right)$   | 74. $\cot\left(-\frac{\pi}{6}\right)$ | 75. $\tan\frac{14\pi}{3}$ | 76. $\sec\frac{11\pi}{4}$              |
| 77. $\sin(8\pi)$   | 78. $\cos(-2\pi)$        | 79. $\tan(7\pi)$   | 80. $\cot(5\pi)$                      | 81. $\sec(-3\pi)$         | 82. $\csc\left(-\frac{5\pi}{2}\right)$ |



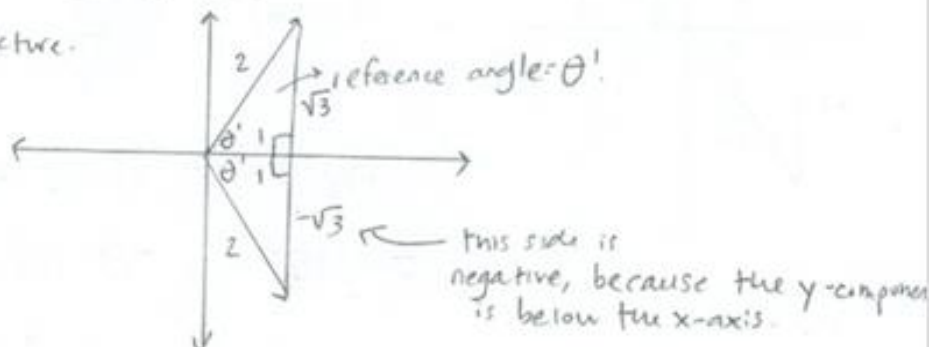
Material

Evaluating inverse trigonometric functions at special angles

Example: Find  $\cos^{-1}\left(\frac{1}{2}\right)$ .

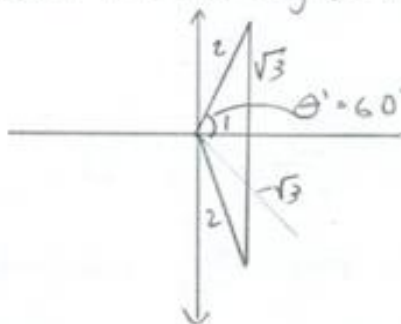
① I know  $\cos(\theta) = \frac{1}{2}$  and  $\frac{1}{2} > 0$ , so  $\theta$  must be in quadrant I or IV.

② Draw a picture.



③ Label sides:  $\frac{\text{adj}}{\text{hyp}} = \frac{1}{2}$ , so adjacent = 1 hypotenuse = 2. } I recognize this as a 30-60-90 triangle. The 3<sup>rd</sup> side is  $\sqrt{3}$ .

④ Label reference angle + the angle opposite the "long side" of a 30-60-90 is  $60^\circ$

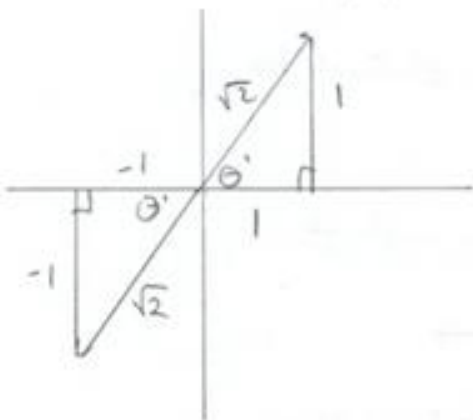


⑤ Find actual angles:

quadrant I:  $60^\circ \cdot \frac{\pi}{180^\circ} = \boxed{\frac{\pi}{3}}$

quadrant IV:  $360^\circ - 60^\circ = 300^\circ \cdot \frac{\pi}{180^\circ} = \boxed{\frac{5\pi}{3}}$

Example:  $\tan^{-1}(1)$



$$\tan(\theta) = 1$$

Positive slope  $\rightarrow \theta$  in quadrant I & III.

$$\tan(\theta) = 1 = \frac{\text{opp}}{\text{adj}} = \frac{1}{1}$$

$$\text{hyp} = \sqrt{2} \rightarrow 45-45-90$$

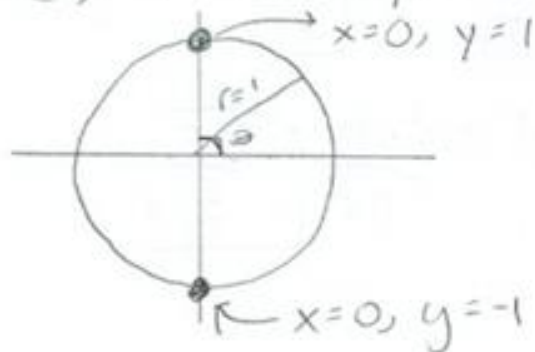
$$\theta' = 45^\circ \text{ so } \theta = 45 \text{ or } \boxed{\frac{\pi}{4}}$$

$$\theta = 180 + 45 = 225$$

$$\text{or } \boxed{\frac{5\pi}{4}}$$

Example:  $\cos^{-1}(\theta) = 0$ .  $\cos \theta = 0 = \frac{\text{adj}}{\text{hyp}} \rightarrow \text{adj} = 0$ .

\*Remember that  $\cos(\theta)$  represents the x coordinate on the unit circle,  $\sin(\theta)$  is the y coordinate, and  $\tan(\theta)$  is the slope.



$$\theta = 90^\circ \text{ or } \boxed{\frac{\pi}{2}}$$

$$\theta = 270^\circ \text{ or } \boxed{\frac{3\pi}{2}}$$

## Practice

For 13–24, find the solutions for all angles  $0 \leq \theta \leq 2\pi$ . For 25–36, find solutions for all angles  $0 \leq \theta \leq 360$ .

## Skill Building

In Problems 13–24, find the exact value of each expression.

- |                          |   |   |   |
|--------------------------|---|---|---|
| 13. $\sin^{-1} 0$        | 14. $\cos^{-1} 1$                               | 15. $\sin^{-1}(-1)$                             | 16. $\cos^{-1}(-1)$                             |
| 17. $\tan^{-1} 0$        | 18. $\tan^{-1}(-1)$                             | 19. $\sin^{-1} \frac{\sqrt{2}}{2}$              | 20. $\tan^{-1} \frac{\sqrt{3}}{3}$              |
| 21. $\tan^{-1} \sqrt{3}$ | 22. $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ | 23. $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ | 24. $\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)$ |

In Problems 25–36, use a calculator to find the value of each expression rounded to two decimal places.

- |                             |                             |                                    |                                    |
|-----------------------------|-----------------------------|------------------------------------|------------------------------------|
| 25. $\sin^{-1} 0.1$         | 26. $\cos^{-1} 0.6$         | 27. $\tan^{-1} 5$                  | 28. $\tan^{-1} 0.2$                |
| 29. $\cos^{-1} \frac{7}{8}$ | 30. $\sin^{-1} \frac{1}{8}$ | 31. $\tan^{-1}(-0.4)$              | 32. $\tan^{-1}(-3)$                |
| 33. $\sin^{-1}(-0.12)$      | 34. $\cos^{-1}(-0.44)$      | 35. $\cos^{-1} \frac{\sqrt{2}}{3}$ | 36. $\sin^{-1} \frac{\sqrt{3}}{5}$ |

## Recognizing graphs of sine and cosine

Table 7

$x$	$y = \sin x$	$(x, y)$
0	0	(0, 0)
$\frac{\pi}{6}$	$\frac{1}{2}$	$\left(\frac{\pi}{6}, \frac{1}{2}\right)$
$\frac{\pi}{2}$	1	$\left(\frac{\pi}{2}, 1\right)$
$\frac{5\pi}{6}$	$\frac{1}{2}$	$\left(\frac{5\pi}{6}, \frac{1}{2}\right)$
$\pi$	0	( $\pi$ , 0)
$\frac{7\pi}{6}$	$-\frac{1}{2}$	$\left(\frac{7\pi}{6}, -\frac{1}{2}\right)$
$\frac{3\pi}{2}$	-1	$\left(\frac{3\pi}{2}, -1\right)$
$\frac{11\pi}{6}$	$-\frac{1}{2}$	$\left(\frac{11\pi}{6}, -\frac{1}{2}\right)$
$2\pi$	0	( $2\pi$ , 0)

The Graph of the Sine Function  $y = \sin x$ 

Since the sine function has period  $2\pi$ , we only need to graph  $y = \sin x$  on the interval  $[0, 2\pi]$ . The remainder of the graph will consist of repetitions of this portion of the graph.

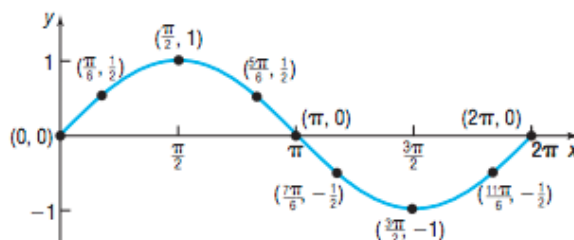
We begin by constructing Table 7, which lists some points on the graph of  $y = \sin x$ ,  $0 \leq x \leq 2\pi$ . As the table shows, the graph of  $y = \sin x$ ,  $0 \leq x \leq 2\pi$ , begins at the origin. As  $x$  increases from 0 to  $\frac{\pi}{2}$ , the value of  $y = \sin x$  increases

from 0 to 1; as  $x$  increases from  $\frac{\pi}{2}$  to  $\pi$  to  $\frac{3\pi}{2}$ , the value of  $y$  decreases from 1 to 0

to -1; as  $x$  increases from  $\frac{3\pi}{2}$  to  $2\pi$ , the value of  $y$  increases from -1 to 0. If we

plot the points listed in Table 7 and connect them with a smooth curve, we obtain the graph shown in Figure 77.

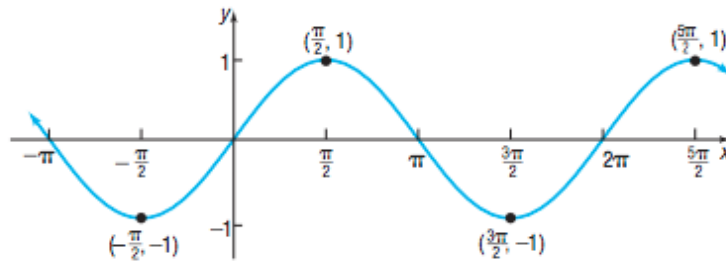
**Figure 77**  
 $y = \sin x$ ,  $0 \leq x \leq 2\pi$



The graph in Figure 77 is one period, or **cycle**, of the graph of  $y = \sin x$ . To obtain a more complete graph of  $y = \sin x$ , we continue the graph in each direction, as shown in Figure 78.

**Figure 78**

$$y = \sin x, -\infty < x < \infty$$



The graph of  $y = \sin x$  illustrates some of the facts that we already know about the sine function.

#### Properties of the Sine Function $y = \sin x$

1. The domain is the set of all real numbers.
2. The range consists of all real numbers from  $-1$  to  $1$ , inclusive.
3. The sine function is an odd function, as the symmetry of the graph with respect to the origin indicates.
4. The sine function is periodic, with period  $2\pi$ .
5. The  $x$ -intercepts are  $\dots, -2\pi, -\pi, 0, \pi, 2\pi, 3\pi, \dots$ ; the  $y$ -intercept is  $0$ .
6. The maximum value is  $1$  and occurs at  $x = \dots, -\frac{3\pi}{2}, \frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}, \dots$ ; the minimum value is  $-1$  and occurs at  $x = \dots, -\frac{\pi}{2}, \frac{3\pi}{2}, \frac{7\pi}{2}, \frac{11\pi}{2}, \dots$

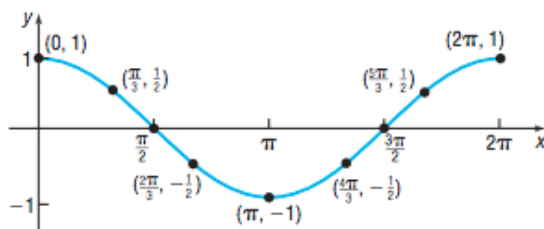
Table 8

$x$	$y = \cos x$	$(x, y)$
0	1	$(0, 1)$
$\frac{\pi}{3}$	$\frac{1}{2}$	$(\frac{\pi}{3}, \frac{1}{2})$
$\frac{\pi}{2}$	0	$(\frac{\pi}{2}, 0)$
$\frac{2\pi}{3}$	$-\frac{1}{2}$	$(\frac{2\pi}{3}, -\frac{1}{2})$
$\pi$	-1	$(\pi, -1)$
$\frac{4\pi}{3}$	$-\frac{1}{2}$	$(\frac{4\pi}{3}, -\frac{1}{2})$
$\frac{3\pi}{2}$	0	$(\frac{3\pi}{2}, 0)$
$\frac{5\pi}{3}$	$\frac{1}{2}$	$(\frac{5\pi}{3}, \frac{1}{2})$
$2\pi$	1	$(2\pi, 1)$

### The Graph of the Cosine Function

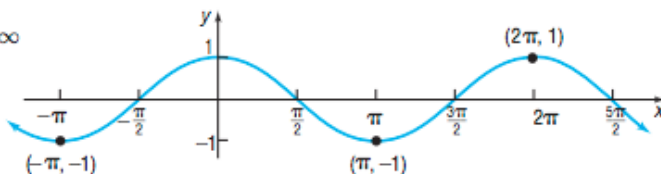
The cosine function also has period  $2\pi$ . We proceed as we did with the sine function by constructing Table 8, which lists some points on the graph of  $y = \cos x$ ,  $0 \leq x \leq 2\pi$ . As the table shows, the graph of  $y = \cos x$ ,  $0 \leq x \leq 2\pi$ , begins at the point  $(0, 1)$ . As  $x$  increases from 0 to  $\frac{\pi}{2}$ , the value of  $y$  decreases from 1 to 0 to  $-1$ ; as  $x$  increases from  $\pi$  to  $\frac{3\pi}{2}$  to  $2\pi$ , the value of  $y$  increases from  $-1$  to 0 to 1. As before, we plot the points in Table 8 to get one period or cycle of the graph. See Figure 81.

**Figure 81**  
 $y = \cos x, 0 \leq x \leq 2\pi$



A more complete graph of  $y = \cos x$  is obtained by continuing the graph in each direction, as shown in Figure 82.

**Figure 82**  
 $y = \cos x, -\infty < x < \infty$



The graph of  $y = \cos x$  illustrates some of the facts that we already know about the cosine function.

#### Properties of the Cosine Function

1. The domain is the set of all real numbers.
2. The range consists of all real numbers from  $-1$  to  $1$ , inclusive.
3. The cosine function is an even function, as the symmetry of the graph with respect to the  $y$ -axis indicates.
4. The cosine function is periodic, with period  $2\pi$ .
5. The  $x$ -intercepts are  $\dots, -\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \dots$ ; the  $y$ -intercept is  $1$ .
6. The maximum value is  $1$  and occurs at  $x = \dots, -2\pi, 0, 2\pi, 4\pi, 6\pi, \dots$ ; the minimum value is  $-1$  and occurs at  $x = \dots, -\pi, \pi, 3\pi, 5\pi, \dots$

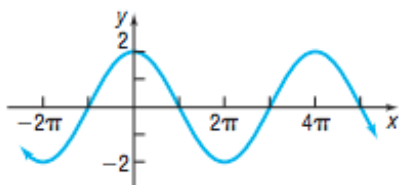
**THEOREM**

If  $\omega > 0$ , the amplitude and period of  $y = A \sin(\omega x)$  and  $y = A \cos(\omega x)$  are given by

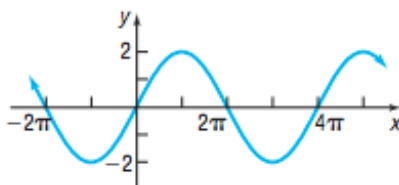
$$\text{Amplitude} = |A| \quad \text{Period} = T = \frac{2\pi}{\omega} \quad (1)$$

**Practice**

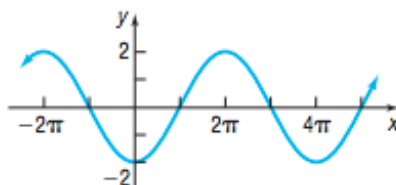
For each of the following, determine if the function is best described as a  $\sin(x)$ ,  $\cos(x)$ ,  $-\sin(x)$ , or  $-\cos(x)$  function.



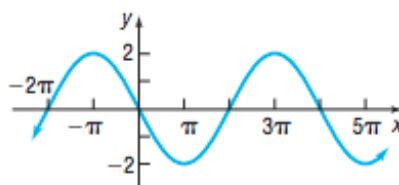
(A)



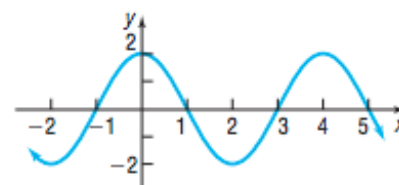
(B)



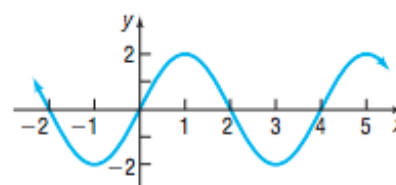
(C)



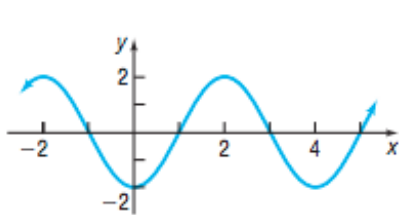
(D)



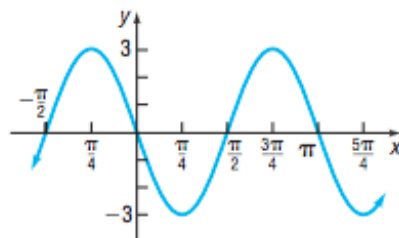
(E)



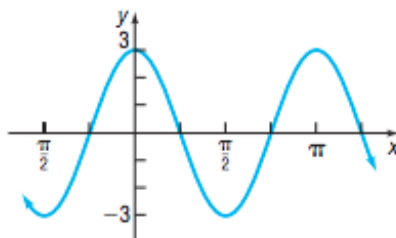
(F)



(G)



(H)



(I)

For each of the following, state the amplitude and the period:

21.  $y = 2 \sin\left(\frac{\pi}{2}x\right)$

22.  $y = 2 \cos\left(\frac{\pi}{2}x\right)$

23.  $y = 2 \cos\left(\frac{1}{2}x\right)$

24.  $y = 3 \cos(2x)$

25.  $y = -3 \sin(2x)$

26.  $y = 2 \sin\left(\frac{1}{2}x\right)$

27.  $y = -2 \cos\left(\frac{1}{2}x\right)$

28.  $y = -2 \cos\left(\frac{\pi}{2}x\right)$

29.  $y = 3 \sin(2x)$

30.  $y = -2 \sin\left(\frac{1}{2}x\right)$

**Sketching graphs of sine and cosine by identifying transformations from the parent function**

Recall that given a function  $y = A\sin(b(x - h)) + k$  or  $y = A\cos(b(x - h)) + k$

We have:

$|A|$  =amplitude

$\frac{2\pi}{b}$  =period

$k$  is the vertical shift, also known as the midline

$h$  is the horizontal shift

The "increment" is equal to the period divided by 4. A sine function goes from the midline  $\rightarrow$  max  $\rightarrow$  midline  $\rightarrow$  minimum  $\rightarrow$  midline over 4 increments (one period). A cosine function goes from the max  $\rightarrow$  midline  $\rightarrow$  minimum  $\rightarrow$  midline  $\rightarrow$  max over 4 increments (one period).

**Practice**

Graph each function. Be sure to label key points and show at least two cycles. It might help to use the template below as you graph each one:

Amplitude:

Max:

Min:

Midline:

Period:

Increment:

Starts at \_\_\_\_\_ (midline, min, max) and goes \_\_\_\_\_ (up or down).

35.  $y = 4 \cos x$

36.  $y = 3 \sin x$

39.  $y = \cos(4x)$

40.  $y = \sin(3x)$

43.  $y = 2 \sin\left(\frac{1}{2}x\right)$

44.  $y = 2 \cos\left(\frac{1}{4}x\right)$

47.  $y = 2 \sin x + 3$

48.  $y = 3 \cos x + 2$

51.  $y = -6 \sin\left(\frac{\pi}{3}x\right) + 4$

52.  $y = -3 \cos\left(\frac{\pi}{4}x\right) + 2$





