Mr. Chadalavada
(615)-815-7622
kchadalavada@kippnashville.org
Office Hours: 10:00 a.m. – 2:00 p.m.

AP Statistics

Learning Packet Overview
This learning packet will mainly focus on Sampling Distributions and Confidence Intervals. There are condensed notes for you to review before attempting the multiple choice and free response problems at the end of each section. Please review this packet over the next two weeks. I will release the answers to the multiple choice and free response portions on 3/30/2020. Please sign up for the AP Statistics-Chadalavada Google classroom with the class code t26t4ur.

Necessary Materials
• You may use calculators, notes, and any resource to review this material. • Khan academy is a good tool for review material on any specific topic • Feel free to reach out to me at any time on questions about these topics.

Confidence Intervals

A **Confidence Interval** is an interval that is computed from sample data and provides a range of plausible values for a population parameter.
A **Confidence Level** is a number that provides information on how much "confidence" we have in the method used to construct a confidence interval estimate. This level specifies the percentage of all possible samples that produce an interval containing the true value of the population parameter.

Constructing a Confidence Interval
The steps listed below should be followed when asked to calculate a confidence interval.

1. Identify the population of interest and define the parameter of interest being estimated.
2. Identify the appropriate confidence interval by name or formula.
3. Verify any conditions (assumptions) that need to be met for that confidence interval.
4. Calculate the confidence interval.
5. Interpret the interval in the context of the situation.

The general formula for a confidence interval calculation is:

Statistic ± (critical value) · (standard deviation of statistic)

The critical value is determined by the confidence level.
The type of statistic is determined by the problem situation. Here we will discuss two types of statistics: mean and proportion.

Confidence Interval for Proportions: (1-proportion z-interval)
We are finding an interval that describes the population proportion (p or π).
The general formula uses the sample proportion ($\hat{p}$) and the sample size (n).

Statistic ± (critical value) · (standard deviation of statistic)

$$\hat{p} \pm (z^*)\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

The conditions that need to be met for this procedure are:
1. The sample is a simple random sample.
2. The population is large relative to the sample.
   $10n < N$ (N = the size of the population)
3. The sampling distribution of the sample proportion is approximately normal.
   $np \geq 10$
   $n(1-p) \geq 10$

Complete the calculations after showing that the conditions are met.
Write the answer in the context of the original problem.

Sample question:

The owner of a popular chain of restaurants wishes to know if completed dishes are being delivered to the customer's table within one minute of being completed by the chef. A random sample of 75 completed dishes found that 60 were delivered within one minute of completion. Find the 95% confidence interval for the true population proportion.

1. Identify the population of interest and define the parameter of interest being estimated.

The population of interest is the dishes that are being served at this chain of restaurants.

p = the population proportion of dishes that are served within one minute of completion.

$\hat{p} = \frac{60}{75} = 0.8$ = the sample proportion of dishes that are served within one minute of completion.

$n = 75$ is the sample size.

2. Identify the appropriate confidence interval by name or formula.

We will use a 95% confidence z-interval for proportions.

3. Verify any conditions (assumptions) that need to be met for that confidence interval.

The problem states that this is a simple random sample.

$10n < N$

$10(75) < N$

$750 < N$

It is reasonable to assume that a popular chain of restaurants will serve more than 750 dishes.

$n\hat{p} \geq 10$ $\quad$ $n(1-\hat{p}) \geq 10$

$75(0.8) = 60$ $\quad$ $75(1-.8) = 15$

$60 \geq 10$ $\quad$ $15 \geq 10$

It is reasonable to use a normal model.

4. Calculate the confidence interval.

At a 95% CI, the critical value is $z^* = 1.96$.

This value is found on the last row of the t-distribution table.

$$0.8 \pm 1.96\sqrt{\frac{0.8(1-0.8)}{75}}$$

$$0.8 \pm 0.091$$

$$(0.709, 0.891)$$

5. Interpret the interval in the context of the situation.

We are 95% confident that the true proportion of dishes that are served within one minute of completion for this chain of restaurants is between 0.709 and 0.891.

<u>Confidence Interval for Means with σ known:</u> (z-interval)
We are finding an interval that describes the population mean (μ).
The general formula uses the sample mean ($\bar{x}$), the population standard deviation (σ) and the sample size (n).

Statistic ± (critical value) · (standard deviation of statistic)

$$\bar{x} \pm \left(z^*\right)\left(\frac{\sigma}{\sqrt{n}}\right)$$

The conditions that need to be met for this procedure are:
1. The sample is a simple random sample.
2. The population is normal or $n \geq 30$
3. The population standard deviation (σ) is known.

Complete the calculations after showing that the conditions are met.
Write your answer in the context of the original problem.

Sample question:

An asbestos removal company places great importance on the safety of their employees. The protective suits that the employees wear are designed to keep asbestos particles off the employee's body. The owner is interested in knowing the average amount of asbestos particles left on the employee's skin after a days work. A random sample of 100 employees had skin tests after removing their protective suit. The average number of particles found was .481 particles per square centimeter. Assuming that the population standard deviation is 0.35 particles per square centimeter, calculate a 95% confidence interval for the number of particles left on the employee's skin.

Solution:

The population of interest is the employees of the asbestos removal company.
μ = the population mean of particles per square centimeter after a days work wearing the protective suit.
$\bar{x} = 0.481$ = the sample mean of the number of particles per square centimeter.
$n = 100$ is the sample size.
$\sigma = 0.35$ =population standard deviation.

We will use a 95% confidence z-interval for means (z-interval).
The problem states that this is a simple random sample.
n = 100 is greater than 30 so we can assume that use of a normal model is reasonable.

At a 95% CI, the critical value is $z^* = 1.96$.
This value is found on the last row of the t-distribution table.

$0.481 \pm 1.96\left(\frac{0.35}{\sqrt{100}}\right)$

0.481 ± 0.069

$(0.412, 0.550)$

We are 95% confident that the true mean number of particles of asbestos found on the skin of an employee after a days work is between 0.412 and 0.550 particles per square centimeter.

<u>Confidence Interval for Means with σ unknown:</u> (t-interval)
You will be finding an interval that will describe the population mean (μ).
The general formula will use the sample mean ($\overline{x}$), the sample standard deviation (s), the sample size (n), and the degrees of freedom ($n-1$).

Statistic ± (critical value) · (standard deviation of statistic)

$$\overline{x} \pm (t^*)\left(\frac{s}{\sqrt{n}}\right)$$

The conditions that need to be met for this procedure are:
1. The sample is a simple random sample.
2. The population is approximately normal (graphical support required) or $n \geq 40$
3. The population standard deviation (σ) is unknown.

Complete the calculations after showing that the conditions are met.
Write your answer in the context of the original problem.

This situation (where the population standard deviation is unknown) is much more realistic that the previous case.

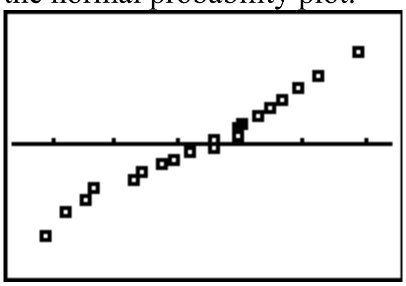
<u>Sample question:</u>
A biology student at a major university is writing a report about bird watchers. She has developed a test that will score the abilities of a bird watcher to identify common birds. She collects data from a random sample of people that classify themselves as bird watchers (data shown below). Find a 90% confidence interval for the mean score of the population of bird watchers.

4.5	9.1	8	5.9	7.0	5.2	7.3	7.0	6.6	5.1
7.6	8.2	6.4	4.8	5.8	6.2	8.5	7.3	7.8	7.4

Solution:

The population of interest is people that classify themselves as bird watchers.
μ = the population mean score on the bird identification ability test.
$\bar{x} = 6.785$ = the sample mean of the scores on the ability test.
$s = 1.2828$ = sample standard deviation of test scores.
$n = 20$ is the sample size.
df = 20 – 1 = 19 degrees of freedom.

We will use a 90% confidence t-interval for means (t-interval).
The problem states that this is a simple random sample.
The sample size, 20, is smaller than 40 so we will assess normality by looking at the normal probability plot.



The normal probability plot appears to be linear so we will assume an approximately normal distribution of scores.
The population standard deviation is unknown.

At a 90% CI, the critical value is $t^* = 1.729$.
This value is found on the t-distribution table using 19 degrees of freedom.

$$6.785 \pm 1.729\left(\frac{1.2828}{\sqrt{20}}\right)$$

$$6.785 \pm 0.49595$$

$$(6.289, 7.281)$$

We are 90% confident that the true mean score on the bird identification ability test of the population of persons that classify themselves as bird watchers is between 6.289 and 7.281.

Summary of Confidence Intervals with One Sample

Confidence Interval Type	Formula	Conditions	Calculator Test
Proportions	$\hat{p} \pm (z^*)\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$	1. The sample is a simple random sample. 2. The population is large relative to the sample $10n < N$ 3. $np \geq 10$ $n(1-p) \geq 10$	1-PropZInterval
Means (σ known)	$\bar{x} \pm (z^*)\left(\frac{\sigma}{\sqrt{n}}\right)$	1. The sample is a simple random sample. 2. The population is normal or $n \geq 30$ 3. The population standard deviation (σ) is known.	ZInterval
Means (σ unknown)	$\bar{x} \pm (t^*)\left(\frac{s}{\sqrt{n}}\right)$	1. The sample is a simple random sample. 2. The population is approximately normal (graphical support required) or $n \geq 40$ 3. The population standard deviation (σ) is unknown.	TInterval

Confidence Intervals with TWO Samples

We may be asked to work a confidence interval problem to estimate the difference of two population parameters. The process is very similar to that of a one sample confidence interval. We must make sure that the conditions are met for both samples and that the samples are independent of each other.

The formulas for the two sample confidence intervals are as follows:

Confidence Interval Type	Formula	Conditions	Calculator Test
Difference of Proportions $(p_1 - p_2)$	$\hat{p}_1 - \hat{p}_2 \pm (z^*)\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$	1. The samples are independently selected simple random samples. 2. Each population is large relative to the sample $10n_1 < N_1$ $10n_2 < N_2$ 3. $n_1 p_1 \geq 10$ $n_1(1-p_1) \geq 10$ $n_2 p_2 \geq 10$ $n_2(1-p_2) \geq 10$	2-PropZInterval
Difference of Means (σ known) $(\mu_1 - \mu_2)$	$\bar{x}_1 - \bar{x}_2 \pm (z^*)\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$	1. The samples are independently selected simple random samples. 2. Both populations are normal or $n_1 \geq 30$ $n_2 \geq 30$ 3. Both population standard deviations (σ_1 and σ_2) are known.	2-samp-ZInterval
Difference of Means (σ unknown) $(\mu_1 - \mu_2)$	$\bar{x}_1 - \bar{x}_2 \pm (t^*)\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$ Note: The formula for degrees of freedom is $df = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{\left(\frac{s_1^2}{n_1}\right)^2}{n_1 - 1} + \frac{\left(\frac{s_2^2}{n_2}\right)^2}{n_2 - 1}}$ This formula is not provided on the formula chart. A more conservative quantity used by some statisticians is the smaller of the two individual degrees of freedom; either $n_1 - 1$ or $n_2 - 1$.	1. The samples are independently selected simple random samples. 2. Both populations are approximately normal (graphical support is required) or $n_1 \geq 40$ $n_2 \geq 40$ 3. Both population standard deviations (σ_1 and σ_2) are unknown.	2-samp-TInterval

Sample question:

Two popular strategy video games, AE and C, are known for their long play times. A popular game review website is interested in finding the mean difference in play time between these games. The website selects a random sample of 43 gamers to play AE and finds their sample mean play time to be 3.6 hours with a standard deviation of 0.9 hours. The website also selected a random sample of gamers to test the game C. There test included 40 gamers with a sample mean of 3.1 hours and a standard deviation of 0.4 hours. Find the 98% confidence interval for the difference $\mu_{AE} - \mu_C$.

Solution:

The population of interest is play time of games AE and C.

μ_{AE} = the population mean play time for game AE.

μ_C = the population mean play time for game C.

$\overline{x}_{AE} = 3.6$ = the sample mean of play time for game AE.

$\overline{x}_C = 3.1$ = the sample mean of play time for game C.

$s_{AE} = 0.9$ = sample standard deviation play time for game AE.

$s_C = 0.4$ = sample standard deviation play time for game C.

$n_{AE} = 43$ is the sample size of the players of game AE.

$n_C = 40$ is the sample size of the players of game C.

df = 43 – 1 = 42 degrees of freedom (using the simplified approximation for df).

We will use a 98% confidence t-interval for the difference of means (2-sample-t-interval).
The problem states that each is a simple random sample.
The sample sizes are 43 and 40 which are both at least 40.
The population standard deviation is unknown.

At a 98% CI, the critical value is $t^* = 2.457$.
This value is found on the t-distribution table using 40 degrees of freedom (because the table does not have a value for 42).

$$\left(3.6 - 3.1\right) \pm 2.423\sqrt{\frac{0.9^2}{43} + \frac{0.4^2}{40}}$$

$$0.5 \pm 0.3662$$

$$(0.1338, 0.8662)$$

We are 98% confident that the true difference in mean play time between games AE and C falls between 0.1338 and 0.8662 hours.

Note: The calculator gives a result of (0.1386, 0.8614) for this confidence interval. The difference in these answers is due to its use of 58.872 degrees of freedom. Our answer has a slightly wider spread, and thus is more conservative.

<u>Other Confidence Interval Topics:</u>

Statistic ± (critical value) · (standard deviation of statistic)

Margin of Error (ME) – the margin of error is the value of the critical value times the standard deviation of the statistic. It is the plus or minus part of the confidence interval.

Some problems might ask you to determine the sample size required given a margin of error. This requires a little algebra to work backward through the equations. The equations are listed below.

Confidence Interval Type	Formula for finding the sample size within a margin of error ME
1-sample proportion	$n = \hat{p}(1-\hat{p})\left(\frac{z^*}{ME}\right)^2$ Note: if $\hat{p}$ is not given in this type of problem, a conservative value to use if 0.5.
1-sample mean (z-interval)	$n = \left(\frac{z^*\sigma}{ME}\right)^2$

Keep in mind the effects of changing the confidence level. A large confidence level (say 99% as compared to 90%) produces a larger margin of error. To be more confident we must include more values in our range.

Do not confuse the meaning of a confidence level: A 95% confidence level means that if we repeated the sampling process many times, the resulting confidence interval would capture the true population parameter 95% of the time.

Multiple Choice Questions on Confidence Intervals

1. A random sample of 100 visitors to a popular theme park spent an average of \$142 on the trip with a standard deviation of \$47.5. Which of the following would the 98% confidence interval for the mean money spent by all visitors to this theme park?

(A) (\$130.77, \$153.23)
(B) (\$132.57, \$151.43)
(C) (\$132.69, \$151.31)
(D) (\$140.88, \$143.12)
(E) (\$95.45, \$188.55)

2. How large of a random sample is required to insure that the margin of error is 0.08 when estimating the proportion of college professors that read science fiction novels with 95% confidence?

(A) 600
(B) 300
(C) 150
(D) 75
(E) 25

3. A quality control specialist at a plate glass factory must estimate the mean clarity rating of a new batch of glass sheets being produced using a sample of 18 sheets of glass. The actual distribution of this batch is unknown, but preliminary investigations show that a normal approximation is reasonable. The specialist decides to use a t-distribution rather than a z-distribution because

(A) The z-distribution is not appropriate because the sample size is too small.
(B) The sample size is large compared to the population size.
(C) The data comes from only one batch.
(D) The variability of the batch is unknown.
(E) The t-distribution results in a narrower confidence interval.

4. An independent random sample of 200 college football players and 150 college basketball players in a certain state showed that 65% of football players received academic tutors while 58% of basketball players received academic tutors. Which of the following is a 90 percent confidence interval for the difference in the proportion of football players that received tutors and the proportion of basketball players that received tutors for the population of this state?

(A) $(.65-.58)\pm 1.96\sqrt{(0.65)(0.58)\left(\frac{1}{200}+\frac{1}{150}\right)}$

(B) $(.65-.58)\pm 1.645\sqrt{(0.65)(0.58)\left(\frac{1}{200}+\frac{1}{150}\right)}$

(C) $(.65-.58)\pm 1.96\sqrt{\frac{(0.65)(0.35)}{200}+\frac{(0.58)(0.42)}{150}}$

(D) $(.65-.58)\pm 1.645\sqrt{\frac{(0.65)(0.35)}{200}+\frac{(0.58)(0.42)}{150}}$

(E) $(.65-.58)\pm 1.645\sqrt{(0.435)(0.867)\left(\frac{1}{200}+\frac{1}{150}\right)}$

5. The board of directors at a city zoo is considering using commercial fast food restaurants in their zoo rather than the current eateries. They are concerned that major donors to the zoo will not approve of the proposed change. Of the 280 major donors to the zoo, a random sample of 90 is asked "Do you support the zoo's decision to use commercial fast food restaurants in the zoo?" 50 of the donors said no, 38 said yes, and 2 had no opinion on the matter. A large sample z-interval, $\hat{p}\pm(z^*)\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$, was constructed from these data to estimate the proportion of the major donors who support using commercial fast food restaurants in the zoo. Which of the following statements is correct for this confidence interval?

(A) This confidence interval is valid because a sample size of more than 30 was used.
(B) This confidence interval is valid because no conditions are required for constructing a large sample confidence interval for proportions.
(C) This confidence interval is not valid because the sample size is too large compared to the population size.
(D) This confidence interval is not valid because the quantity $n\hat{p}$ is too small.
(E) This confidence interval is not valid because "no opinion" was allowed as a response.

6. A research and development engineer is preparing a report for the board of directors on the battery life of a new cell phone they have produced. At a 95% confidence level, he has found that the battery life is 3.2 ± 1.0 days. He wants to adjust his findings so the margin of error is as small as possible. Which of the following will produce the smallest margin of error?

(A) Increase the confidence level to 100%. This will assure that there is no margin of error.
(B) Increase the confidence level to 99%.
(C) Decrease the confidence level to 90%.
(D) Take a new sample from the population using the exact same sample size.
(E) Take a new sample from the population using a smaller sample size.

7. A biologist has taken a random sample of a specific type of fish from a large lake. A 95 percent confidence interval was calculated to be 6.8 ± 1.2 pounds. Which of the following is true?

(A) 95 percent of all the fish in the lake weigh between 5.6 and 8 pounds.
(B) In repeated sampling, 95 percent of the sample proportions will fall within 5.6 and 8 pounds.
(C) In repeated sampling, 95% of the time the true population mean of fish weights will be equal to 6.8 pounds.
(D) In repeated sampling, 95% of the time the true population mean of fish weight will be captured in the constructed interval.
(E) We are 95 percent confident that all the fish weigh less than 8 pounds in this lake.

8. A polling company is trying to estimate the percentage of adults that consider themselves happy. A confidence interval based on a sample size of 360 has a larger than desired margin of error. The company wants to conduct another poll and obtain another confidence interval of the same level but reduce the error to one-third the size of the original sample. How many adults should they now interview?

(A) 40
(B) 180
(C) 720
(D) 1080
(E) 3240

9. A researcher is interested in determining the mean energy consumption of a new compact florescent light bulb. She takes a random sample of 41 bulbs and determines that the mean consumption is 1.3 watts per hour with a standard deviation of 0.7. When constructing a 97% confidence interval, which would be the most appropriate value of the critical value?

(A) 1.936
(B) 2.072
(C) 2.250
(D) 2.704
(E) 2.807

10. A 98 percent confidence interval for the mean of a large population is found to be 978 ± 25. Which of the following is true?

(A) 98 percent of all observations in the population fall between 953 and 1003
(B) The probability of randomly selecting an observation between 953 and 1003 from the population is 0.98
(C) If the true population mean is 950, then this sample mean of 978 would be unlikely to occur.
(D) If the true population mean is 990, then this sample mean of 978 would be unexpected.
(E) If the true population mean is 1006, then this confidence interval must have been calculated incorrectly.

Free Response Questions on Confidence Intervals

Free Response 1.

A random sample of 9th grade math students was asked if they prefer working their math problems using a pencil or a pen. Of the 250 students surveyed, 100 preferred pencil and 150 preferred pen.

(a) Using the results of this survey, construct a 95 percent confidence interval for the proportion of 9th grade students that prefer to work their math problems in pen.

(b) A school newspaper reported on the results of this survey by saying, "Over half of ninth-grade math students prefer to use pen on their math assignments." Is this statement supported by your confidence interval? Explain.

Free Response 2.

A major city in the United States has a large number of hotels. During peak travel times throughout the year, these hotels use a higher price for their rooms. A travel agent is interested in finding the difference of the average cost of a hotel rooms from the peak season to the off season. He takes a random sample of hotel room costs during each of these seasons. Plots of both samples of data indicate that the assumption of normality is not unreasonable.

Season	Cost	Standard Deviation	Sample Size
Peak Season	$245	$45	33
Off Season	$135	$65	38

(a) Construct a 95 percent confidence interval for the difference of the mean cost of hotel rooms from peak season to the off season.

(b) One particular hotel has an off season rate of $88 and a peak season rate of $218. Based on your confidence interval, comment on the price difference of this hotel.

Finding probabilities using a normal model (forwards!)

- Strategy: $x \rightarrow z \rightarrow P$.
- Sketch a normal model, draw a vertical line at the x value, and shade the area of interest.
- Take the x value and find the z score using the formula.
- Then look up the z score on the normal table to find the probability below (to the left of) the line. (See the last page for calculator usage)
- If you want to find the area to the right of the line, subtract the P value off the table from 1.
- If you want to find the area between two lines, find the z scores for each x value, look up P values on the table for each z score, and subtract the P values.

Finding an x value given a normal probability (backwards!)

- Strategy is $P \rightarrow z \rightarrow x$.
- Sketch a normal model, draw a vertical line where you think the x value will be, and label the area/probability you have been given.
- Remember, the table lists the area to the left of the line. If you have been given an area to the right, subtract from 1 to get the area on the left.
- Find the P value inside the chart and move outwards to read the z score.
- Plug the z-score as well as the mean and standard deviation into the formula and solve for x.

Sampling Distributions and The Central Limit Theorem

Consider taking many (theoretically, all possible) samples of size n from a population. Take the average $\bar{x}$ of each sample. All of these sample means make up the sampling distribution, which can be graphed as a histogram.

- The mean of the sampling distribution is the same as the mean of the population: $\mu_{\bar{x}} = \mu$
- The standard deviation of the sampling distribution gets smaller according to this equation: $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$
- The Central Limit Theorem states that as the sample size n increases, the sampling distribution becomes more normal (regardless of the shape of the population). In practice, if $n \geq 30$, we assume the distribution is approximately normal.

When do you use the Central Limit Theorem?

Use the Central Limit Theorem when a question asks you to calculate a probability about an average or mean.

Example:
The amount of dirt loaded into a dump truck varies normally with a mean of 750 pounds and a standard deviation of 40 pounds. A) Find the probability that a randomly chosen truck would hold over 800 pounds of dirt. B) Find the probability that the average weight in a random sample of 5 trucks would be over 800 pounds.

Solution:

A) x = weight of dirt in a dump truck

x follows a $N(750,40)$ model

$$z = \frac{800-750}{40} = 1.25,\ P(x > 800)) = P(z > 1.25) = 1 - .8944 = .1056$$

Nearly 11% of the trucks (10.56%) would hold over 800 pounds.

B) $\bar{x}$ = average weight of dirt in a sample of 5 dump trucks

$\bar{x}$ follows a $N\left(750, \frac{40}{\sqrt{5}} = 17.89\right)$ model

$$z = \frac{800-750}{17.89} = 2.80$$

$$P(\bar{x} > 800)) = P(z > 2.80) = 1 - .9974 = .0026$$

Less than one percent (0.26%) of groups of 5 dump trucks would have an average weight over 800 pounds.

Multiple Choice Questions on Normal Models and Sampling Distributions

1. If heights of 3rd graders follow a normal distribution with a mean of 52 inches and a standard deviation of 2.5 inches, what is the z score of a 3rd grader who is 47 inches tall?

(A) -5
(B) -2
(C) 2
(D) 5
(E) 26.2

2. Suppose that a normal model describes the acidity (pH) of rainwater, and that water tested after last week's storm had a *z*-score of 1.8. This means that the acidity of the rain

(A) had a pH 1.8 higher than average rainfall.
(B) had a pH of 1.8.
(C) varied with standard deviation 1.8
(D) had a pH 1.8 standard deviations higher than that of average rainwater.
(E) had a pH 1.8 times that of average rainwater.

3. In a factory, the weight of the concrete poured into a mold by a machine follows a normal distribution with a mean of 1150 pounds and a standard deviation of 22 pounds. Approximately 95% of molds filled by this machine will hold weights in what interval?

(A) 1084 to 1216 pounds
(B) 1106 to 1150 pounds
(C) 1106 to 1194 pounds
(D) 1128 to 1172 pounds
(E) 1150 to 1194 pounds

4. Which of the following are true?

I. In a normal distribution, the mean is always equal to the median.
II. All unimodal and symmetric distributions are normal for some value of μ and σ.
III. In a normal distribution, nearly all of the data is within 3 standard deviations of the mean, no matter the mean and standard deviation.

(A) I only
(B) II only
(C) III only
(D) I and III only
(E) I, II, and III

5. The height of male Labrador Retrievers is normally distributed with a mean of 23.5 inches and a standard deviation of 0.8 inches. (The height of a dog is measured from his shoulder.) Labradors must fall under a height limit in order to participate in certain dog shows. If the maximum height is 24.5 inches for male labs, what percentage of male labs are not eligible?

(A) 0.1056
(B) 0.1250
(C) 0.8750
(D) 0.8944
(E) 0.9750

6. The heights of mature pecan trees are approximately normally distributed with a mean of 42 feet and a standard deviation of 7.5 feet. What proportion of pecan trees are between 43 and 46 feet tall?

(A) 0.1501
(B) 0.2969
(C) 0.4470
(D) 0.5530
(E) 0.7031

7. Heights of fourth graders are normally distributed with a mean of 52 inches and a standard deviation of 3.5 inches. Ten percent of fourth graders should have a height below what number?

(A) -1.28 inches
(B) 45.0 inches
(C) 47.5 inches
(D) 48.9 inches
(E) 56.5 inches

8. A large college class is graded on a total points system. The total points earned in a semester by the students in the class vary normally with a mean of 675 and a standard deviation of 50. Another large class in a different department is graded on a 0 to 100 scale. The final grades in that class follow a normal model with a mean of 82 and a standard deviation of 6. Jessica earns 729 points in the first class, while Ana scores 90 in the second class. Which student did better and why?

(A) Jessica did better because her score is 54 points above the mean while Ana's is only 8 points above the mean
(B) The students did equally well because both scored above the mean.
(C) Ana did better because her score is 1.33 standard deviations above the mean while Jessica's is only 1.08 standard deviations above the mean.
(D) Neither student did better; they cannot be compared because their classes have different scoring systems.

9. The distance Jonathan can throw a shot put is skewed to the right with a mean of 14.2 meters and a standard deviation of 3.5 meters. Over the course of a month, Jonathan makes 75 throws during practice. Assume these throws can be considered a random sample of Jonathan's shot put throws. What is the probability that Jonathan's average shot put distance for the month will be over 15.0 meters?

(A) 0.0239
(B) 0.4096
(C) 0.5224
(D) 0.5904
(E) 0.9761

10. Heights of fourth graders are normally distributed with a mean of 52 inches and a standard deviation of 3.5 inches. For a research project, you plan to measure a simple random sample of 30 fourth graders. For samples such as yours, 10% of the samples should have an average height below what number?

(A) 47.52 inches
(B) 51.18 inches
(C) 51.85 inches
(D) 52.82 inches
(E) 56.48 inches

<u>Free Response Questions on Normal Models and Sampling Distributions</u>

Free Response 1.
A machine is used to fill soda bottles in a factory. The bottles are labeled as containing 2.0 liters, but extra room at the top of the bottle allows for a maximum of 2.25 liters of soda before the bottle overflows. The standard deviation of the amount of soda put into the bottles by the machine is known to be 0.15 liter.

(a) Overfilling the bottles causes a mess on the assembly line, but consumers will complain if bottles contain less than 2 liters. If the machine is set to fill the bottles with an average of 2.08 liters, what proportion of bottles will be overfilled?

(b) If management requires that no more than 3% of bottles should be overfilled, the machine should be set to fill the bottles with what mean amount?

(c) Complaints from consumers about underfilled bottles leads the company to set the mean amount to 2.15 liters. In this situation, what standard deviation would allow for no more than 3% of the bottles to be overfilled?

Free Response 2.
The distribution of scores for persons over 16 years of age on a common IQ test is approximately normal with mean 100 and standard deviation 15.

a) What is the probability that a randomly chosen adult has an IQ score on this test over 105?

b) What are the mean and standard deviation of the average IQ score on this test for an SRS of 60 people?

c) What is the probability that the average IQ score on this test of an SRS of 60 people is 105 or higher?

d) Would your *method* of answering (a) or (c) be affected if the distribution of IQ scores on this test in the adult population were distinctly nonnormal (for example, if they were skewed)? Explain which parts could be answered the same way, which could not, and how you know.