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AP Calculus AB

Learning Packet Overview
This learning packet will provide an alternative/supplement to the review resources available via google classroom. We will primarily be utilizing google classroom, AP classroom, and Khan Academy for review. This packet will provide review work that can replace or supplement online review.

Necessary Materials
• Pencil/paper • TI-84 graphing calculator

How students will be successful in AP Calculus	How caregivers can help students be successful
Students will be successful if: • Students devote 30+ min/day to reviewing for the AP exam • Students complete the problems under named time/calculator conditions • Students review correct answers when available • Students reach out to classmates and/or to Ms. Heath for help early and often!	Caregivers can help students be successful if they: • Encourage students to commit 30+ minutes to review each day • Cheer on students who are doing difficult, college level math! • Encourage students to reach out to Ms. Heath early and often – if they are struggling with a problem for more than 10 minutes, ask for help! • Create a distraction free environment: focus on one thing at a time (if doing school work, put away social media, turn off the TV, etc)

Practice Problems

*If possible, check google classroom daily for updated assignments.

For preliminary review, please see the attached "Calc Cheat Sheet" document. For the next two days, spend 30 min/day creating flash cards for these key formulas/derivative rules. Knowing the contents of this document by heart will be crucial for the AP exam! Plan to use the next two days to create the notecards, and then plan to review the notecards for 5 min 2-3x/day between now and the AP exam.

The rest of this document includes **5 practice FRQ questions** along with scoring guides for your review. A graphing calculator is required for questions 1-2, but not permitted on questions 3-5.

A GRAPHING CALCULATOR IS REQUIRED FOR THESE QUESTIONS.

h (feet)	0	2	5	10
$A(h)$ (square feet)	50.3	14.4	6.5	2.9

1. A tank has a height of 10 feet. The area of the horizontal cross section of the tank at height h feet is given by the function A, where $A(h)$ is measured in square feet. The function A is continuous and decreases as h increases. Selected values for $A(h)$ are given in the table above.

   (a) Use a left Riemann sum with the three subintervals indicated by the data in the table to approximate the volume of the tank. Indicate units of measure.

   (b) Does the approximation in part (a) overestimate or underestimate the volume of the tank? Explain your reasoning.

   (c) The area, in square feet, of the horizontal cross section at height h feet is modeled by the function f given by $f(h) = \dfrac{50.3}{e^{0.2h} + h}$. Based on this model, find the volume of the tank. Indicate units of measure.

   (d) Water is pumped into the tank. When the height of the water is 5 feet, the height is increasing at the rate of 0.26 foot per minute. Using the model from part (c), find the rate at which the volume of water is changing with respect to time when the height of the water is 5 feet. Indicate units of measure.

2. When a certain grocery store opens, it has 50 pounds of bananas on a display table. Customers remove bananas from the display table at a rate modeled by

$$f(t) = 10 + (0.8t)\sin\left(\frac{t^3}{100}\right) \text{ for } 0 < t \leq 12,$$

where $f(t)$ is measured in pounds per hour and t is the number of hours after the store opened. After the store has been open for three hours, store employees add bananas to the display table at a rate modeled by

$$g(t) = 3 + 2.4\ln\left(t^2 + 2t\right) \text{ for } 3 < t \leq 12,$$

where $g(t)$ is measured in pounds per hour and t is the number of hours after the store opened.

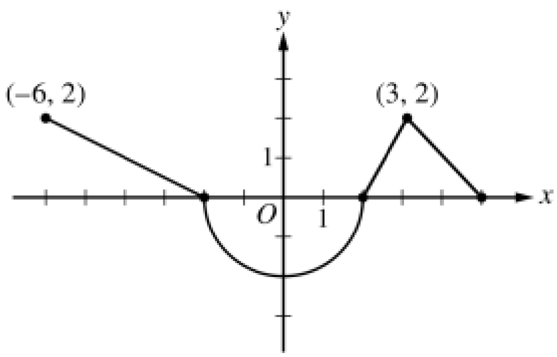
(a) How many pounds of bananas are removed from the display table during the first 2 hours the store is open?

(b) Find $f'(7)$. Using correct units, explain the meaning of $f'(7)$ in the context of the problem.

(c) Is the number of pounds of bananas on the display table increasing or decreasing at time $t = 5$? Give a reason for your answer.

(d) How many pounds of bananas are on the display table at time $t = 8$?

NO CALCULATOR IS ALLOWED FOR THESE QUESTIONS.



Graph of f'

3. The function f is differentiable on the closed interval $[-6, 5]$ and satisfies $f(-2) = 7$. The graph of f', the derivative of f, consists of a semicircle and three line segments, as shown in the figure above.

(a) Find the values of $f(-6)$ and $f(5)$.

(b) On what intervals is f increasing? Justify your answer.

(c) Find the absolute minimum value of f on the closed interval $[-6, 5]$. Justify your answer.

(d) For each of $f''(-5)$ and $f''(3)$, find the value or explain why it does not exist.

4. At time $t = 0$, a boiled potato is taken from a pot on a stove and left to cool in a kitchen. The internal temperature of the potato is 91 degrees Celsius (°C) at time $t = 0$, and the internal temperature of the potato is greater than 27°C for all times $t > 0$. The internal temperature of the potato at time t minutes can be modeled by the function H that satisfies the differential equation $\frac{dH}{dt} = -\frac{1}{4}(H - 27)$, where $H(t)$ is measured in degrees Celsius and $H(0) = 91$.

(a) Write an equation for the line tangent to the graph of H at $t = 0$. Use this equation to approximate the internal temperature of the potato at time $t = 3$.

(b) Use $\frac{d^2H}{dt^2}$ to determine whether your answer in part (a) is an underestimate or an overestimate of the internal temperature of the potato at time $t = 3$.

(c) For $t < 10$, an alternate model for the internal temperature of the potato at time t minutes is the function G that satisfies the differential equation $\frac{dG}{dt} = -(G - 27)^{2/3}$, where $G(t)$ is measured in degrees Celsius and $G(0) = 91$. Find an expression for $G(t)$. Based on this model, what is the internal temperature of the potato at time $t = 3$?

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5. Two particles move along the x-axis. For $0 \le t \le 8$, the position of particle P at time t is given by $x_P(t) = \ln\left(t^2 - 2t + 10\right)$, while the velocity of particle Q at time t is given by $v_Q(t) = t^2 - 8t + 15$. Particle Q is at position $x = 5$ at time $t = 0$.

(a) For $0 \le t \le 8$, when is particle P moving to the left?

(b) For $0 \le t \le 8$, find all times t during which the two particles travel in the same direction.

(c) Find the acceleration of particle Q at time $t = 2$. Is the speed of particle Q increasing, decreasing, or neither at time $t = 2$? Explain your reasoning.

(d) Find the position of particle Q the first time it changes direction.

Scoring Guidelines

Question 1

	1 : units in parts (a), (c), and (d)
(a) Volume $= \int_0^{10} A(h)\,dh$ $\approx (2-0)\cdot A(0) + (5-2)\cdot A(2) + (10-5)\cdot A(5)$ $= 2\cdot 50.3 + 3\cdot 14.4 + 5\cdot 6.5$ $= 176.3$ cubic feet	2 : $\begin{cases} 1 : \text{left Riemann sum} \\ 1 : \text{approximation} \end{cases}$
(b) The approximation in part (a) is an overestimate because a left Riemann sum is used and A is decreasing.	1 : overestimate with reason
(c) $\int_0^{10} f(h)\,dh = 101.325338$ The volume is 101.325 cubic feet.	2 : $\begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$
(d) Using the model, $V(h) = \int_0^h f(x)\,dx$. $\left.\frac{dV}{dt}\right\|_{h=5} = \left[\frac{dV}{dh}\cdot\frac{dh}{dt}\right]_{h=5}$ $= \left[f(h)\cdot\frac{dh}{dt}\right]_{h=5}$ $= f(5)\cdot 0.26 = 1.694419$ When $h = 5$, the volume of water is changing at a rate of 1.694 cubic feet per minute.	3 : $\begin{cases} 2 : \frac{dV}{dt} \\ 1 : \text{answer} \end{cases}$

Question 2

(a) $\int_0^2 f(t)\,dt = 20.051175$ 20.051 pounds of bananas are removed from the display table during the first 2 hours the store is open.	$2: \begin{cases} 1: \text{integral} \\ 1: \text{answer} \end{cases}$
(b) $f'(7) = -8.120$ (or -8.119) After the store has been open 7 hours, the rate at which bananas are being removed from the display table is decreasing by 8.120 (or 8.119) pounds per hour per hour.	$2: \begin{cases} 1: \text{value} \\ 1: \text{meaning} \end{cases}$
(c) $g(5) - f(5) = -2.263103 < 0$ Because $g(5) - f(5) < 0$, the number of pounds of bananas on the display table is decreasing at time $t = 5$.	$2: \begin{cases} 1: \text{considers } f(5) \text{ and } g(5) \\ 1: \text{answer with reason} \end{cases}$
(d) $50 + \int_3^8 g(t)\,dt - \int_0^8 f(t)\,dt = 23.347396$ 23.347 pounds of bananas are on the display table at time $t = 8$.	$3: \begin{cases} 2: \text{integrals} \\ 1: \text{answer} \end{cases}$

Question 3

(a) $f(-6) = f(-2) + \int_{-2}^{-6} f'(x)\,dx = 7 - \int_{-6}^{-2} f'(x)\,dx = 7 - 4 = 3$

$f(5) = f(-2) + \int_{-2}^{5} f'(x)\,dx = 7 - 2\pi + 3 = 10 - 2\pi$

3 : { 1 : uses initial condition; 1 : $f(-6)$; 1 : $f(5)$ }

(b) $f'(x) > 0$ on the intervals $[-6, -2)$ and $(2, 5)$.
Therefore, f is increasing on the intervals $[-6, -2]$ and $[2, 5]$.

2 : answer with justification

(c) The absolute minimum will occur at a critical point where $f'(x) = 0$ or at an endpoint.

$f'(x) = 0 \Rightarrow x = -2, x = 2$

x	$f(x)$
-6	3
-2	7
2	$7 - 2\pi$
5	$10 - 2\pi$

The absolute minimum value is $f(2) = 7 - 2\pi$.

2 : { 1 : considers $x = 2$; 1 : answer with justification }

(d) $f''(-5) = \dfrac{2 - 0}{-6 - (-2)} = -\dfrac{1}{2}$

$$\lim_{x \to 3^-} \frac{f'(x) - f'(3)}{x - 3} = 2 \text{ and } \lim_{x \to 3^+} \frac{f'(x) - f'(3)}{x - 3} = -1$$

$f''(3)$ does not exist because

$$\lim_{x \to 3^-} \frac{f'(x) - f'(3)}{x - 3} \neq \lim_{x \to 3^+} \frac{f'(x) - f'(3)}{x - 3}.$$

2 : { 1 : $f''(-5)$; 1 : $f''(3)$ does not exist, with explanation }

Question 4

(a) $H'(0) = -\frac{1}{4}(91-27) = -16$ $H(0) = 91$ An equation for the tangent line is $y = 91 - 16t$. The internal temperature of the potato at time $t = 3$ minutes is approximately $91 - 16 \cdot 3 = 43$ degrees Celsius.	3 : { 1 : slope 1 : tangent line 1 : approximation
(b) $\frac{d^2H}{dt^2} = -\frac{1}{4}\frac{dH}{dt} = \left(-\frac{1}{4}\right)\left(-\frac{1}{4}\right)(H-27) = \frac{1}{16}(H-27)$ $H > 27$ for $t > 0 \Rightarrow \frac{d^2H}{dt^2} = \frac{1}{16}(H-27) > 0$ for $t > 0$ Therefore, the graph of H is concave up for $t > 0$. Thus, the answer in part (a) is an underestimate.	1 : underestimate with reason
(c) $\frac{dG}{(G-27)^{2/3}} = -dt$ $\int \frac{dG}{(G-27)^{2/3}} = \int(-1)\,dt$ $3(G-27)^{1/3} = -t + C$ $3(91-27)^{1/3} = 0 + C \Rightarrow C = 12$ $3(G-27)^{1/3} = 12 - t$ $G(t) = 27 + \left(\frac{12-t}{3}\right)^3$ for $0 \le t < 10$ The internal temperature of the potato at time $t = 3$ minutes is $27 + \left(\frac{12-3}{3}\right)^3 = 54$ degrees Celsius.	5 : { 1 : separation of variables 1 : antiderivatives 1 : constant of integration and uses initial condition 1 : equation involving G and t 1 : $G(t)$ and $G(3)$ Note: max $2/5$ [1-1-0-0-0] if no constant of integration Note: $0/5$ if no separation of variables

Question 5

(a) $x_P'(t) = \dfrac{2t-2}{t^2-2t+10} = \dfrac{2(t-1)}{t^2-2t+10}$

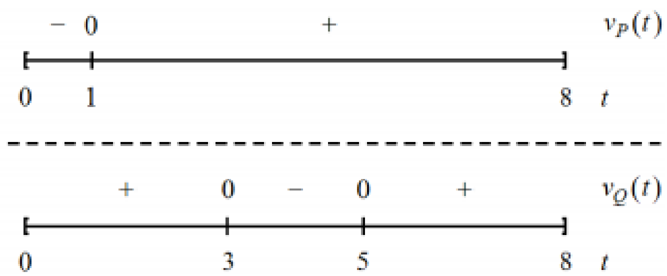
$t^2 - 2t + 10 > 0$ for all t.

$x_P'(t) = 0 \Rightarrow t = 1$
$x_P'(t) < 0$ for $0 \le t < 1$.

Therefore, the particle is moving to the left for $0 \le t < 1$.

$2 : \begin{cases} 1 : x_P'(t) \\ 1 : \text{interval} \end{cases}$

(b) $v_Q(t) = (t-5)(t-3)$
$v_Q(t) = 0 \Rightarrow t = 3, t = 5$



Both particles move in the same direction for $1 < t < 3$ and $5 < t \le 8$ since $v_P(t) = x_P'(t)$ and $v_Q(t)$ have the same sign on these intervals.

$2 : \begin{cases} 1 : \text{intervals} \\ 1 : \text{analysis using } v_P(t) \text{ and } v_Q(t) \end{cases}$

Note: $1/2$ if only one interval with analysis

Note: $0/2$ if no analysis

(c) $a_Q(t) = v_Q'(t) = 2t - 8$
$a_Q(2) = 2 \cdot 2 - 8 = -4$

$a_Q(2) < 0$ and $v_Q(2) = 3 > 0$

At time $t = 2$, the speed of the particle is decreasing because velocity and acceleration have opposite signs.

$2 : \begin{cases} 1 : a_Q(2) \\ 1 : \text{speed decreasing with reason} \end{cases}$

(d) Particle Q first changes direction at time $t = 3$.

$$x_Q(3) = x_Q(0) + \int_0^3 v_Q(t)\,dt = 5 + \int_0^3 \left(t^2 - 8t + 15\right) dt$$
$$= 5 + \left[\frac{1}{3}t^3 - 4t^2 + 15t\right]_{t=0}^{t=3} = 5 + (9 - 36 + 45) = 23$$

$3 : \begin{cases} 1 : \text{antiderivative} \\ 1 : \text{uses initial condition} \\ 1 : \text{answer} \end{cases}$